Generation of zonal magnetic fields by low-frequency dispersive electromagnetic waves in a nonuniform dusty magnetoplasma

P. K. Shukla
Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
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It is shown that zonal magnetic fields can be parametrically excited by low-frequency dispersive driftlike compressional electromagnetic (DDCEM) modes in a nonuniform dusty magnetoplasma. For this purpose, we derive a pair of coupled equations which exhibits the nonlinear coupling between DDCEM modes and zonal magnetic fields. The coupled mode equations are Fourier analyzed to derive a nonlinear dispersion relation. The latter depicts that zonal magnetic fields are nonlinearly generated at the expense of the low-frequency DDCEM wave energy. The relevance of our investigation to the transfer of energy from short scale DDCEM waves to long scale zonal magnetic field structures in dark molecular clouds is discussed.

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I. INTRODUCTION

Several years ago Birk *et al.* [1] developed the magnetohydrodynamic equations for dusty plasmas and studied the magnetic reconnection processes that lead to the formation of magnetic islands. Three years ago, Rudakov *et al.* [2] dwelled on the role of charged dust particles in decoupling of the plasma from the magnetic field in a nonuniform astrophysical plasma. The decoupling of a nonuniform dusty plasma (containing electrons, ions, and stationary charged dust grains) from the magnetic field is attributed to the existence of a low-frequency (in comparison with the ion gyrofrequency) nondispersive magnetic dust drift wave [2,3], which is associated with the rotational electric field [1] $\mathbf{E}=(\mathbf{J}\times\mathbf{B})/cq_d n_{d0}$, where $\mathbf{J}=e(n_{i0}\mathbf{v}_i-n_{e0}\mathbf{v}_e)=(c/4\pi)\nabla\times\mathbf{B}$ is the sum of the ion and electron current densities, $\mathbf{B}$ is the sum of the dc and oscillating magnetic fields, $\mathbf{v}_i(\mathbf{v}_e)$ is the ion (electron) fluid velocity, $q_d=-ez_d(ez_d)$ for negatively (positively) charged dust grains, e is the magnitude of the electron charge, z_d is the number of charges on dust grains, n_{j0} is the unperturbed number density of the particle species j (j equals e for electrons, i for ions, and d for dust grains), and c is the speed of light in vacuum. In the magnetic dust drift waves, both the electrons and ions suffer the $c\mathbf{E}\times\mathbf{B}/B^2$ drift, and in a nonuniform dusty plasma we thus have $\mathbf{J}\approx-(q_d n_{d0}ec/B^2)\mathbf{E}\times\mathbf{B}$. The frequency of the magnetic dust drift wave [2] is proportional [inversely proportional] to the magnetic field strength B_0 [to $\partial(z_d n_{d0})^{-1}/\partial x$]. This mode is also contained in Eq. (25) of Ref. [1] when the plasma has the dust density inhomogeneity. Rudakov [3] also illuminated the properties of magnetic sound and whistler waves, and reported a low-frequency (in comparison with the ion gyrofrequency) magnetic monopole vortex whose dynamics is governed by the Hasegawa-Mima equation [4]. Shukla and Mamun [5] discussed the properties of magnetic shock waves that are based on Eq. (5) of Rudakov [3]. Furthermore, Rudakov [6] applied the Hall dynamo physics to astrophysical dusty plasmas. An up-to-date knowledge of the magnetohydrodynamic waves in dusty plasmas is contained in two textbooks [7,8].

Very recently, Shukla [9,10] reported a new low-frequency [in comparison with the Rao cutoff frequency [11] $\Omega_R=z_d n_{d0}\omega_{ci}/n_{e0}$, where $\omega_{ci}=eB_0/m_i c$ is the ion gyrofrequency and m_i is the ion mass] compressional electromagnetic wave in a nonuniform magnetoplasma composed of electrons, ions, and an ensemble of stationary charged dust grains. The presence of the latter in a magnetoplasma produces charge imbalance and an associated electric field, which in conjunction with the $\mathbf{J}\times\mathbf{B}$ force causes acceleration and rotation of ions [11]. When the ion rotation frequency Ω_R (or the Rao cutoff frequency [11]) is much larger than the wave frequency under consideration, the cross-field ion drift velocity in the compressional magnetic field perturbation is inversely proportional to the dust charge density. We thus have a unique ion motion in the plane perpendicular to the external magnetic field $B_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the z direction, Since in an inhomogeneous dusty magnetoplasma the electric field is rotational, there appears a dispersive driftlike compressional electromagnetic (DDCEM) mode [9,10] propagating in a direction perpendicular to both the density gradient and external magnetic field directions.

In this Brief Report, we show that finite amplitude DDCEM modes can parametrically excite large scale zonal magnetic fields in a nonuniform dusty magnetoplasma. Physically, a short wavelength (in comparison with $\rho=\lambda_i n_{i0}/z_d n_{d0}$, where $\lambda_i=c/\omega_{pi}$ is the ion skin depth and $\omega_{pi}=\sqrt{4\pi n_{i0}e^2/m_i}$ is the ion plasma frequency) DDCEM mode interacts nonlinearly with long wavelength zonal magnetic field perturbations and generates short wavelength DDCEM sidebands. The latter, in turn, interact with the DDCEM pump and produce the Reynolds stress on zonal magnetic fields, which are amplified. Such a nonlinear mechanism can thus produces large scale magnetic field structures [12], which are relevant for astrophysical settings that are composed of charged dust grains [13–15], electrons and ions.

II. GOVERNING EQUATIONS

We consider a multicomponent magnetoplasma composed of neutrals, electrons, ions, and charged dust grains. The external magnetic field is $B_0\hat{\mathbf{z}}$. The neutrals and micron-sized charged dust grains are supposed to be stationary. The plasma has a density gradient $\partial n_{j0}/\partial x$ along the x axis. At equilibrium, we have $en_{i0}(x)=en_{e0}(x)-q_d(x)n_{d0}(x)$. We shall focus on the dynamics of low-frequency (in comparison with Ω_R) electromagnetic waves in the $x-y$ plane by isolating the motion of electrons and ions parallel to $\hat{\mathbf{z}}$. The governing equations for our purposes are [10]

$$(d_i+\nu_{in})\mathbf{v}_i=\frac{(\nabla\times\mathbf{B})\times\mathbf{B}}{4\pi n_e m_i}+\frac{q_d n_d}{c n_e m_i}\mathbf{v}_i\times\mathbf{B}, \tag{1}$$

and

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}_e\times\mathbf{B}), \tag{2}$$

where $d_t=\partial/\partial t+\mathbf{v}_i\cdot\nabla$, ν_{in} is the ion-neutral collision frequency, $\mathbf{B}=\hat{\mathbf{z}}(B_0+B_1)$ is the total magnetic field, $B_1 \equiv B_1(x,y,t)$ is the compressional magnetic field perturbation, $n_i(n_e)$ is the total ion (electron) number density, and the electron-neutral collision frequency is supposed to be smaller than the electron gyrofrequency. The electron fluid velocity is

$$\mathbf{v}_e=\frac{n_i}{n_e}\mathbf{v}_i-\frac{c}{4\pi e n_e}\nabla\times\mathbf{B}, \tag{3}$$

which is obtained from the Maxwell equation by neglecting the displacement current, in view of the low phase speed (in comparison with c) electromagnetic waves under consideration. In the absence of dust, we have $n_i=n_e$ and $\Omega_R=0$. Consequently, the second term in the right-hand side of Eq. (1) identically vanishes.

In the following, we consider a dusty plasma ($n_{i0}\neq n_{e0}$) and focus on $|d_t+\nu_{in}|\ll\Omega_R$ and $B_1\ll B_0$. Thus, we have from Eq. (1)

$$\mathbf{v}_{i\perp}\approx\frac{c}{4\pi q_d n_{d0}}\hat{\mathbf{z}}\times\nabla_\perp B_1-\frac{n_{e0}m_i c^2}{4\pi q_d^2 n_{d0}^2 B_0}(D_t+\nu_{in})\nabla_\perp B_1, \tag{4}$$

where the subscript $\perp$ stands for the component perpendicular to $\hat{\mathbf{z}}$, and we have denoted

$$D_t=\frac{\partial}{\partial t}+\frac{c}{4\pi q_d n_{d0}}\hat{\mathbf{z}}\times\nabla B_1\cdot\nabla_\perp,$$

and

$$\nabla_\perp=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},0\right).$$

Equation (4) exhibits that the oscillatory ion fluid velocity in the compressional magnetic field perturbation B_1 is inversely proportional to the dust charge density.

Combining Eqs. (2)–(4), and taking the z component of the resulting equation, we obtain to leading order [10],

$$\partial_t B_1-\frac{cB_0}{4\pi}\hat{\mathbf{z}}\times\nabla\left(\frac{1}{q_d n_{d0}}\right)\cdot\nabla B_1-\rho^2(D_t+\nu_{in})\nabla_\perp^2 B_1=0, \tag{5}$$

which generalizes the work of Shukla [9] by including the ion-neutral drag (the ν_{in} term). In the linear regime, we Fourier analyze Eq. (5) to obtain the frequency of damped DDCEM modes [10],

$$\omega=\frac{k_y U_*}{1+k_\perp^2\rho^2}-i\frac{\nu_{in}k_\perp^2\rho^2}{1+k_\perp^2\rho^2}, \tag{6}$$

where k_y is the y component of the wave vector, $U_*=-(cB_0/4\pi)\,\partial(1/q_d n_{d0})/\partial x$, and $k_\perp^2=k_x^2+k_y^2$.

On the other hand, zonal magnetic fields, which have $k_y=0$, are purely damped due to ion-neutral collisions, i.e.,

$$\omega=-i\frac{\nu_{in}k_x^2\rho^2}{1+k_x^2\rho^2}, \tag{7}$$

which shows that long wavelength (in comparison with ρ) zonal magnetic fields will have a long lifetime. Zonal magnetic fields can be excited by the DDCEM modes, as discussed below.

The nonlinear interactions between the DDCEM modes and zonal magnetic fields are governed by

$$\frac{\partial}{\partial t}(1-\rho^2\nabla_\perp^2)B_{1s}+U_*\frac{\partial B_{1s}}{\partial y}-D\nabla_\perp^2 B_{1s}-\frac{c\rho^2}{4\pi q_d n_{d0}}[(\hat{\mathbf{z}}\times\nabla B_{1z}\cdot\nabla)\nabla_\perp^2 B_{1s}+(\hat{\mathbf{z}}\times\nabla B_{1s}\cdot\nabla)\nabla_\perp^2 B_{1z}]=0, \tag{8}$$

which is obtained by setting $B_1=B_{1s}+B_{1z}$ in Eq. (5). Here $D=\nu_{in}\rho^2$, and the subscript s and z represent the quantities associated with the DDCEM modes and zonal magnetic fields, respectively.

For azimuthally symmetric zonal magnetic fields, we obtain from Eq. (5)

$$\frac{\partial}{\partial t}(1-\rho^2\nabla_\perp^2)B_{1z}-D\nabla_\perp^2 B_{1z}-\frac{c\rho^2}{4\pi q_d n_{d0}}\times\langle(\hat{\mathbf{z}}\times\nabla B_{1s}\cdot\nabla)\nabla_\perp^2 B_{1s}\rangle=0, \tag{9}$$

where the fourth term in the left-hand side is the averaged (over the DDCEM wave period) Reynolds stress of rapidly oscillating DDCEM modes.

III. THE NONLINEAR DISPERSION RELATION

Nonlinear interactions between a finite amplitude DDCEM pump $(\omega_0,\mathbf{k}_0)$ and zonal magnetic winds $(\Omega,\mathbf{K})$ excite upper and lower sidebands $(\omega_\pm,\mathbf{k}_\pm)$, where $\omega_\pm=\Omega\pm\omega_0$ and $\mathbf{k}_\pm=\mathbf{K}\pm\mathbf{k}_0$ are the frequencies and wave vectors of the sidebands. Thus, we write

$$B_{1s}=B_{s0+}\exp(-i\omega_0 t+i\mathbf{k}_0\cdot\mathbf{r})+B_{s0-}\exp(i\omega_0 t-i\mathbf{k}_0\cdot\mathbf{r})+\sum_{+,-}B_{1s\pm}\exp(-i\omega_\pm t+i\mathbf{k}_\pm\cdot\mathbf{r}), \tag{10a}$$

and

$$B_{1z}=\hat{B}_{1z}\exp(-i\Omega t+i\mathbf{K}\cdot\mathbf{r}), \qquad (10b)$$

where the subscript $0\pm$ and $\pm$ stand for the pump and sidebands, respectively.

Inserting Eqs. (10a) and (10b) into Eqs. (8) and (9) and Fourier analyzing we obtain

$$S_{\pm}B_{1s\pm}=\pm i\frac{cq_{\perp}^2\rho^2}{4\pi q_d n_{d0}}\frac{\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{K}}{(1+k_{\perp\pm}^2\rho^2)}B_{s0\pm}\hat{B}_{1z}, \qquad (11)$$

and

$$(\Omega+i\Gamma_z)\hat{B}_{1z}=i\frac{c\rho^2}{4\pi q_d n_{d0}}\frac{\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{K}}{(1+K^2\rho^2)}\times(K_-^2B_{s0+}B_{1s-}-K_+^2B_{s0-}B_{1s+}), \qquad (12)$$

where $S_{\pm}=\omega_{\pm}-\omega_{*\pm}+i\Gamma_{s\pm}$, $\omega_{*\pm}=k_{y\pm}U_*/\alpha_{\pm}$, $\Gamma_{s\pm}=\nu_{in}k_{\perp\pm}^2\rho^2/\alpha_{\pm}$, $\alpha_{\pm}=1+k_{\perp\pm}^2\rho^2$, $q_{\perp}^2=K_{\perp}^2-k_{\perp0}^2$, $K_{\pm}^2=k_{\perp\pm}^2-k_0^2$, and $\Gamma_z=\nu_{in}K^2\rho^2/(1+K^2\rho^2)$. Equation (11) shows that the DDCEM sidebands are created due to the beating of the DDCEM pump and zonal magnetic fields. The latter, in turn, are amplified due to the low-frequency ponderomotive force arising from the DDCEM pump and sidebands, as depicted in the right-hand side of Eq. (12).

Eliminating $B_{1s\pm}$ from Eq. (12) by using Eq. (11) we obtain the nonlinear dispersion relation

$$\Omega+i\Gamma_z=\frac{c^2\rho^4B_{s0}^2}{16\pi^2q_d^2n_{d0}^2}\frac{|\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{K}|^2q_{\perp}^2}{(1+K_{\perp}^2\rho^2)}\sum_{+,-}\frac{K_{\pm}^2}{\alpha_{\pm}S_{\pm}}, \qquad (13)$$

where $B_{s0}^2=B_{s0+}B_{s0-}$. In the absence of the pump, we have $\Omega=-i\Gamma_z$, which is the damping rate of zonal magnetic flows.

For $|\Omega|\gg\Gamma_z$, $\omega_0\gg\Gamma_{s\pm}$, and $k_{\perp0}^2\gg K_{\perp}^2$ we have from Eq. (13)

$$\Omega^2\approx-\frac{k_{\perp0}^2c^2\rho^4B_{s0}^2}{8\pi^2q_d^2n_{d0}^2}\frac{|\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{K}|^2}{(1+K_{\perp}^2\rho^2)}\frac{\mathbf{k}_0\cdot\mathbf{K}_{\perp}}{(1+k_{\perp0}^2\rho^2)}, \qquad (14)$$

which depicts a purely growing ($\Omega=i\Omega_i$) instability when the DDCEM pump is turned on. For $\mathbf{k}_0\cdot\mathbf{K}_{\perp}>0$, the growth rate for the azimuthally symmetric zonal magnetic field excitation is

$$\Omega_i=\frac{k_{\perp0}c\rho^2B_{s0}}{2\sqrt{2}\pi z_d e n_{d0}}\frac{|\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{K}||\mathbf{k}_{0\perp}\cdot\mathbf{K}_{\perp}|^{1/2}}{[(1+K_{\perp}^2\rho^2)(1+k_{\perp0}^2\rho^2)]^{1/2}}, \qquad (15)$$

which reveals that the increment of the purely growing zonal magnetic fields is directly proportional to the magnetic field amplitude of the DDCEM pump. The e-folding time (Ω_i^{-1}) of the present instability for dark molecular clouds, where $n_{i0}\sim\times10^{-4}$ cm^{-3}, $n_{d0}\sim10^{-7}$ cm^{-3}, $z_d\sim10$, $B_0\sim230$ μG is roughly 10^4 s for $k_{\perp0}\rho\sim1$, $|\mathbf{K}_{\perp}|\sim0.1|\mathbf{k}_{\perp0}|$, and $B_{s0}\sim23\mu$G. The growth time is shorter than the ion-neutral relaxation rate, which is 2×10^5 s for the neutral density of roughly 10^4 cm^{-3}.

IV. SUMMARY AND CONCLUSION

In summary, we have presented a mechanism for generating long wavelength (in comparison with ρ) zonal magnetic fields by low-frequency (in comparison with Ω_R), short wavelength DDCEM modes in a nonuniform dusty magnetoplasma. Specifically, it has been shown that energy from short wavelength DDCEM mode can be parametrically coupled to sidebands and long wavelength zonal magnetic fields. The latter are amplified due to the Reynolds stress arising from the beating of the DDCEM pump and sidebands. The growth rate for the zonal magnetic field excitation is directly proportional to the magnetic field strength of the DDCEM pump. Parametrically excited zonal magnetic fields attain large amplitudes and may self-organize in the form of large scale vortices. Such vortices constitute a dynamical paradigm for intermittency in interstellar media containing nonlinearly coupled DDCEM modes and zonal magnetic fields. Hence, the present investigation provides an essential nonlinear mechanism for the transfer of energy from low-frequency short wavelength DDCEM modes to large scale enhanced zonal magnetic winds in nonuniform interstellar dusty magnetoplasmas [13–15] whose constituents are neutrals, electrons, ions, and charged dust grains. Our results may also be useful in understanding the origin of low-frequency compressional electromagnetic wave driven zonal magnetic winds in low-temperature laboratory dusty plasma discharges. We are hoping that forthcoming laboratory experiments in an external magnetic field should be able to verify our theoretical prediction of DDCEM waves in association with zonal magnetic fields.

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[1] G. T. Birk, A. Kopp, and P. K. Shukla, Phys. Plasmas **3**, 3564 (1996).
[2] see L. I. Rudakov, A. V. Gretchikha, C. S. Liu, and G. M. Milikh, http://arxiv.org/ftp/astro-ph/papers/0002/0002293.pdf
[3] L. I. Rudakov, Phys. Scr. **T89**, 158 (2001).
[4] A. Hasegawa and K. Mima, Phys. Fluids **21**, 87 (1978).
[5] P. K. Shukla and A. A. Mamun, Phys. Scr. **64**, 351 (2001).
[6] L. I. Rudakov, Phys. Scr. **T98**, 58 (2002).
[7] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer Academic, Dordrecht, 2000).

[8] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002).
[9] P. K. Shukla, Phys. Lett. A **316**, 238 (2003).
[10] P. K. Shukla, Phys. Plasmas **10**, 4907 (2003).
[11] N. N. Rao, J. Plasma Phys. **53**, 317 (1995).
[12] B. G. Elmegreen, Astrophys. J. **515**, 323 (1999); **530**, 277 (2000).
[13] H. Kayama and R. Nishi, Astrophys. J. **543**, 257 (2000).
[14] A. Lazarian and H. Yan, Astrophys. J. **566**, L105 (2002).
[15] H. Yan and A. Lazarian, Astrophys. J. **592**, L33 (2003).